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Scalar flavour-changing neutral currents in the large-$\tan\beta$ limit

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Abstract: We analyse scalar flavour-changing neutral currents of down-type quarks in models with two Higgs doublets, coupled separately to up- and down-type quarks, in the limit where the ratio of the two expectation values ($\tan\beta = v_u/v_d$) is large. We clarify the origin of this phenomenon, both in $\Delta F = 1$ and $\Delta F = 2$ processes, analysing differences and analogies between supersymmetric and non-supersymmetric models. We confirm previous findings of a sizeable enhancement at large $\tan\beta$ of specific $\Delta F = 1$ and $\Delta F = 2$ amplitudes in the MSSM and, in these cases, we discuss how large-$\tan\beta$ corrections can be controlled beyond lowest order. Finally, we emphasize the unique role of the rare processes $B_{s,d} \to \tau^+\tau^-$ and $B_{s,d} \to \mu^+\mu^-$ in probing this scenario.

Keywords: Beyond Standard Model, Supersymmetric Standard Model, Rare Decays.

JHEP11(2001)001

Contents

1. Introduction

Processes mediated by flavour-changing neutral-current (FCNC) amplitudes are extremely useful to deeply investigate the dynamics of quark-flavour mixing: the strong suppression of these transitions occurring within the standard model (SM), due to the absence of tree-level contributions and the hierarchy of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, naturally enhance their sensitivity to possible non-standard phenomena. This is even more true in the case of down-type FCNC amplitudes mediated by scalar (and pseudoscalar) currents that, within the SM, are additionally suppressed by the smallness of down-type Yukawa couplings.

The suppression of down-type FCNC scalar operators becomes less effective in models with an extended Higgs sector and, particularly, in the popular two Higgs doublets model (2HDM) of type II, where two $\mathrm{SU}(2)_L$ scalar doublets are coupled separately to up- and down-type quarks (see e.g. ref. [1]). In this case FCNC amplitudes are still absent at the tree level. However, contrary to the SM, it is possible to accommodate large down-type Yukawa couplings, provided the ratio $v_u/v_d = \tan\beta$, where v_u (v_d) denotes the vacuum expectation value of the doublet coupled to up (down)-type quarks, is large. Then, for instance, $\Delta F = 1$ operators like $\bar{b}_R s_L \bar{\mu}_R \mu_L$, suppressed by two small Yukawa couplings, could receive a $\tan^2\beta$ enhancement with respect to the SM case.

Even though the $\tan^2\beta$ enhancement could be as large as 10^3 in the simple 2HDM, scalar FCNC amplitudes barely reach the level of vector-type SM amplitudes, and this happens only in very few cases. On the other hand, the 2HDM of type II is particularly interesting being the Higgs sector of the Minimal Supersymmetric SM (MSSM) (see e.g. refs. [1, 2]). As first noted by Babu and Kolda [3], and later confirmed in [4, 5], the $\tan\beta$ enhancement of scalar FCNC amplitudes can be much more effective in the MSSM than in the non-supersymmetric case. In particular, the one-loop coefficient of the operator $\bar{b}_R s_L \bar{\mu}_R \mu_L$ has been found to scale like $\tan^3\beta$, leading to possible large non-standard effects in $B_s \to \mu^+\mu^-$. More recently, it has also been shown that the coefficient of the $\Delta F = 2$ operator $\bar{b}_R s_L \bar{b}_L s_R$ receives, at the two-loop level, a contribution scaling as $\tan^4\beta$ that could have a relevant impact on B_s–$\bar{B}_s$ mixing [6].

The purpose of this paper is to clarify the nature of this $\tan\beta$ enhancements. We will analyse the large-$\tan\beta$ behaviour of all relevant down-type $\Delta F = 1$ and $\Delta F = 2$ amplitudes, both in supersymmetric and non-supersymmetric cases. As already pointed out in [3], the large-$\tan\beta$ enhancement of scalar FCNC amplitudes is intimately related to the appearance, at the one-loop level, of an effective coupling between H_u and down-type quarks [7]. As we shall show, this is a necessary consequence of any 2HDM and indeed it is realized in a very similar way in the supersymmetric and in the non-supersymmetric case. The difference between the two scenarios arises only under a specific conspiracy of the soft-breaking terms, which could decouple the supersymmetric $\widetilde{H}_u$–$\widetilde{H}_d$ mixing from the ordinary H_u–H_d coupling, necessarily suppressed in the large-$\tan\beta$ limit. Analysing both scenarios, we shall clarify the origin of the different $\tan\beta$ factors, showing how to control the large-$\tan\beta$ terms beyond lowest order.

The paper is organized as follows. Section 2 is devoted to $\Delta F = 1$ amplitudes; there we shall first analyse the generic structure of the effective $d_i - d_j - H^0$ vertex, then we shall discuss the large-$\tan\beta$ behaviour of $d_i \to d_j \ell^+\ell^-$ amplitudes. In section 3 we analyse $B_{s,d}$–$\bar{B}_{s,d}$ mixing, discussing the generic structure of both reducible and one-particle irreducible contributions and deriving phenomenological bounds from $\Delta M_{s,d}$. Section 4 contains a phenomenological analysis of the rare decays $B_{s,d} \to \tau^+\tau^-$ and $B_{s,d} \to \mu^+\mu^-$. The results are summarized in the conclusions.

2. $\Delta F = 1$ scalar currents

2.1 The effective down-type Yukawa interaction

In a 2HDM of type II (including the MSSM) the tree-level Yukawa interaction is defined as

$$\mathcal{L}_Y^0 = \overline{d}_R Y_d Q_L H_d + \overline{u}_R Y_u Q_L H_u \ + \ \text{h.c.}\,, \tag{2.1}$$

where $Y_{u,d}$ are 3×3 matrices in flavour space and $H_{u,d}$ denote the two Higgs doublets. This Lagrangian is invariant under a global U(1) symmetry, which we shall call U$(1)_d$ and under which $\overline{d}_R$ and H_d have opposite charge and all the other fields are neutral. If this symmetry were exact, the coupling of H_u to down-type quarks would be forbidden also at the quantum level. This symmetry, however, is naturally broken by terms appearing in the Higgs potential and, if $\tan\beta$ is large, this has a substantial impact on the effective Yukawa interaction of down-type quarks. Under the assumption that L_Y^0 is the only source of flavour mixing and that the model-dependent Higgs self-couplings have the MSSM structure [1, 2], the one-loop effective down-type Yukawa lagrangian (in both the supersymmetric and non-supersymmetric cases) can be written as [7, 8, 3]:

$$\mathcal{L}_d^{\text{eff}} = \overline{d}_R Y_d \left[H_d + \left(\epsilon_0 + \epsilon_Y Y_u^\dagger Y_u\right) H_u^* \right] Q_L \ + \ \text{h.c.}\,, \tag{2.2}$$

where $\epsilon_{0,Y}$ denote appropriate loop functions $[\epsilon_{0,Y} \sim \mathcal{O}(1/16\pi^2)]$ proportional to the U$(1)_d$-breaking terms.

In order to diagonalize the mass terms generated by $\mathcal{L}_d^{\text{eff}}$, it is convenient to rotate the quark fields in the basis where Y_d is diagonal $[(Y_d)_{ij} = y_i^d \delta_{ij}]$. In this basis we can write

$$\mathcal{L}_{d-\text{mass}}^{\text{eff}} = v_d \overline{d}_R^i y_i^d \left[(1+\epsilon_0 \tan\beta)\delta_{ij} + \epsilon_Y \tan\beta \; V_{ik}^{0\dagger} (y_k^u)^2 V_{kj}^0 \right] d_L^j \ + \ \text{h.c.}\,, \tag{2.3}$$

where $v_d^2(1+\tan^2\beta) = (2\sqrt{2}G_F)^{-1} \approx (174~\text{GeV})^2$ and V^0 denotes the tree-level CKM matrix, i.e. the CKM matrix in the limit $\epsilon_Y = 0$. Neglecting the small terms due to $y_{1,2}^u$, defining $\lambda_{jk}^t = V_{j3}^{0\dagger} V_{3k}^0$ (for $j \neq k$) and $y_t = y_3^u$, we can rewrite eq. (2.3) as

$$\mathcal{L}_{d-\text{mass}}^{\text{eff}} = v_d \overline{d}_R^i \bar{y}_i^d \left[\delta_{ij} + \frac{\epsilon_Y y_t^2 \tan\beta}{1+\tan\beta\,(\epsilon_0 + \epsilon_Y y_t^2 |V_{i3}^0|^2)} \lambda_{ij}^t \right] d_L^j \ + \ \text{h.c.}\,, \tag{2.4}$$

where

$$\bar{y}_i^d = y_i^d \left[1 + \tan\beta\left(\epsilon_0 + \epsilon_Y y_t^2 |V_{i3}^0|^2\right)\right] \ \approx \ y_i^d \left[1+\tan\beta\left(\epsilon_0 + \epsilon_Y y_t^2 \delta_{i3}\right)\right]. \tag{2.5}$$

Because of the hierarchy of the CKM matrix, $\lambda_{ij}^t \ll 1$ and we can diagonalize eq. (2.4) perturbatively in λ_{ij}^t. The rotation that diagonalizes eq. (2.4) to the first order in

JHEP11(2001)001

λ^t_{ij} is given by

$$d^j_L \to \left[\delta_{jk} + \epsilon_Y y_t^2 \tan\beta \frac{\bar{y}^d_j y^d_j + \bar{y}^d_k y^d_k}{(\bar{y}^d_k)^2 - (\bar{y}^d_j)^2} \lambda^t_{jk}\right] d^k_L\,, \tag{2.6}$$

$$d^j_R \to \left[\delta_{jk} + \epsilon_Y y_t^2 \tan\beta \frac{\bar{y}^d_j y^d_k + \bar{y}^d_k y^d_j}{(\bar{y}^d_k)^2 - (\bar{y}^d_j)^2} \lambda^t_{jk}\right] d^k_R\,. \tag{2.7}$$

At this order the eigenvalues are not shifted, and thus the leading $\tan\beta$ corrections to the eigenvalues of the down-type Yukawa matrix are simply described by eq. (2.5) [7]. On the other hand, the rotation (2.6) modifies the structure of the CKM matrix, but only the V_{3j} ($j \neq 3$) elements receive $\mathcal{O}(1)$ corrections (i.e. corrections not suppressed by the CKM hierarchy) [8]:

$$\frac{V_{3j}}{V^0_{3j}} = \frac{V_{j3}}{V^0_{j3}} = \frac{1+\epsilon_0 \tan\beta}{1+\tan\beta(\epsilon_0 + \epsilon_Y y_t^2)}\,. \tag{2.8}$$

As first noted by Babu and Kolda [3], if $\epsilon_Y \neq 0$ the diagonalization of eq. (2.4) necessarily induces a FCNC coupling between quarks and neutral Higgs bosons. This can easily be understood by looking at the neutral component of eq. (2.2): if $\epsilon_Y \neq 0$ and Y_u is not aligned to Y_d, we have two independent flavour structures (not simultaneously diagonalizable) that are weighted differently for physical Higgses and mass terms. Indeed performing explicitly the rotations (2.6) and (2.7) and neglecting the subleading $\mathcal{O}(y^d_j/y^d_{k>j})$ terms, we find

$$\mathcal{L}^{\rm eff}_{{\rm FCNC}\ (k\neq j)} = y^d_k \bar{\lambda}^t_{kj} \frac{\epsilon_Y y_t^2 \tan\beta}{1+\epsilon_0 \tan\beta} \left[\frac{1}{\tan\beta} H^{0*}_u - H^0_d\right] \overline{d}^k_R d^j_L \ +\ {\rm h.c.}\,, \tag{2.9}$$

where, expressing the off-diagonal coupling in terms of the physical CKM matrix,

$$\bar{\lambda}^{t*}_{jk} = \bar{\lambda}^t_{kj} = \begin{cases} V^*_{33} V_{3j} & (k=3)\,, \\ V^*_{k3} V_{3j} \left[\frac{1+\tan\beta(\epsilon_0+\epsilon_Y y_t^2)}{1+\epsilon_0 \tan\beta}\right]^2 & (k \neq 3)\,. \end{cases} \tag{2.10}$$

Equation (2.9) generalizes to any $d^k \to d^j$ transition the result obtained by Babu and Kolda for the $b \to s$ case.

One of the most interesting aspects of eq. (2.9) is the rapid growth with $\tan\beta$ of the $H^0_d \overline{d}^k_R d^j_L$ coupling. Expressing y^d_k in terms of down-type quark masses, the FCNC coupling grows almost quadratically in $\tan\beta$, contrary to the approximate linear growth of the diagonal $H^0_d \overline{d}^k_R d^k_L$ term. This is the origin of the approximate $\tan^3\beta$ and $\tan^4\beta$ behaviour of the $\Delta F = 1$ and $\Delta F = 2$ amplitudes discussed in [3] and [6], respectively.

As expected, the combination of neutral Higgs fields that appears in eq. (2.9) has a vanishing vacuum expectation value and does not include the Goldstone component; it can therefore be expressed in terms of the three physical neutral Higgs

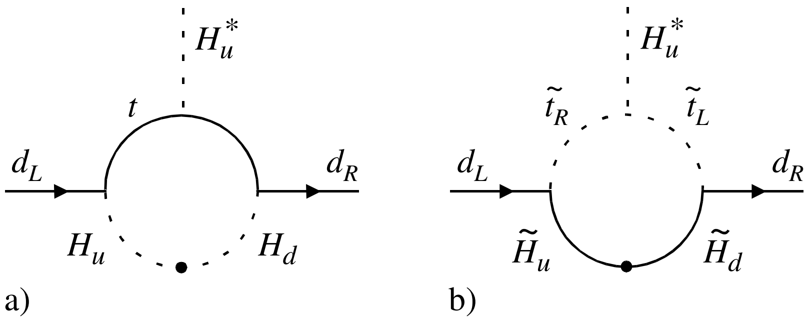


Figure 1: Penguin diagrams generating a non-vanishing ϵ_Y: a) non-supersymmetric H_u–H_d mixing; b) chargino mixing.

states. Assuming the Higgs potential to be CP-invariant and employing the notation of [2] we find

$$\mathcal{L}^{\rm eff}_{{\rm FCNC}\ (k\neq j)} = y^d_k \bar{\lambda}^t_{kj} \frac{\epsilon_Y y_t^2}{\sqrt{2}\cos\beta[1+\epsilon_0\tan\beta]} \times \\ \times \left[\cos(\alpha-\beta)h^0 + \sin(\alpha-\beta)H^0 - iA^0\right] \overline{d}^k_R d^j_L \ + \ \text{h.c.} \quad (2.11)$$

2.2 Explicit estimates of ϵ_Y

In the context of a non-supersymmetric 2HDM with Higgs self-couplings fixed as in the MSSM case, the only source of $\mathrm{U}(1)_d$ breaking is the bilinear operator $H_u H_d$. If the Higgs potential is CP-invariant, the coupling of this operator, conventionally denoted by μB, is related to the mass of the neutral pseudoscalar Higgs by the (tree-level) relation

$$\mu B = M_A^2 \sin 2\beta/2\,. \quad (2.12)$$

Thus $\mathrm{U}(1)_d$ breaking is unavoidable if we require a non-vanishing mass for A^0. Note, however, that in this case the $\mathrm{U}(1)_d$ breaking is parametrically suppressed in the large-$\tan\beta$ limit.

The non-supersymmetric H_u–H_d mixing induces a non-vanishing ϵ_Y via the mechanism shown in figure 1a. The computation of this effect at large $\tan\beta$ can be performed by setting $g = 0$, i.e. switching off gauge interactions: keeping $g \neq 0$ would have complicated the calculation, owing to the gauge-dependent mixing of charged Higgs fields and W bosons, without affecting the final result. In this limit we can identify H_u^+ with the massless Goldstone boson and H_d^+ with the massive charged Higgs, and the only diagram to be computed is the one in figure 1a, which leads to

$$\epsilon_Y^{\rm 2HDM} = \frac{1}{16\pi^2}\frac{\mu B}{M^2_{H^+}}\left[\log\left(m_t^2/M^2_{H^+}\right) + \mathcal{O}\left(m_t^2/M^2_{H^+}\right)\right]. \quad (2.13)$$

In the supersymmetric case an additional source of $\mathrm{U}(1)_d$ breaking is provided by the $\mu\widetilde{H}_u\widetilde{H}_d$ term. The latter contribute to ϵ_Y via the diagram of figure 1b, leading to [7]:

$$\epsilon_Y^{\rm SUSY} = \frac{1}{16\pi^2}\frac{\mu A}{M^2_{\tilde{t}_L}} f\left(x_{\mu L}, x_{RL}\right), \tag{2.14}$$

where, as usual, A denotes the coupling of the soft-breaking trilinear term (we assume both μ and A to be real), $x_{\mu L} = \mu^2/M^2_{\tilde{t}_L}$ and $x_{RL} = M^2_{\tilde{t}_R}/M^2_{\tilde{t}_L}$. The full expression of $f(x,y)$ can be found in the appendix and the normalization is such that $f(1,1) = 1/2$. In the supersymmetric case one finds also a non-vanishing ϵ_0, dominated by the contribution of gluino penguins [7].

As can be noted, the explicit expressions of $\epsilon_Y^{\rm 2HDM}$ and $\epsilon_Y^{\rm SUSY}$ are rather similar: $\epsilon_Y \sim 1/(16\pi^2)$ times an adimensional coupling parametrizing the $\mathrm{U}(1)_d$ breaking. In the non-supersymmetric case the $\mathrm{U}(1)_d$ breaking is strongly constrained by eq. (2.12), which forces ϵ_Y to be suppressed as $1/\tan\beta$ in the large-$\tan\beta$ limit. Thus the potential $\tan^2\beta$ growth of the $H_d^0 \overline{d}^i_R d^j_L$ coupling cannot be realized in the simple 2HDM model. On the contrary, in the supersymmetric case we are allowed to consider a scenario where $\mu A/M^2_{\tilde{t}_L} = \mathcal{O}(1)$ also at large $\tan\beta$. Note, however, that this scenario implies a sizeable hierarchy among soft-breaking terms: if $M_{\tilde{t}_L} \gtrsim M_A$ we need $A/B \gtrsim \tan\beta$, while if $A/B \lesssim 1$ we need $M^2_{\tilde{t}_L}/M^2_A \lesssim \tan\beta$.

2.3 $d_i \to d_j \ell^+\ell^-$ transitions

We are now ready to discuss the effect of scalar-current amplitudes in $d_i \to d_j \ell^+\ell^-$ transitions. The effective Hamiltonian describing these processes, including scalar-current operators, can be written as

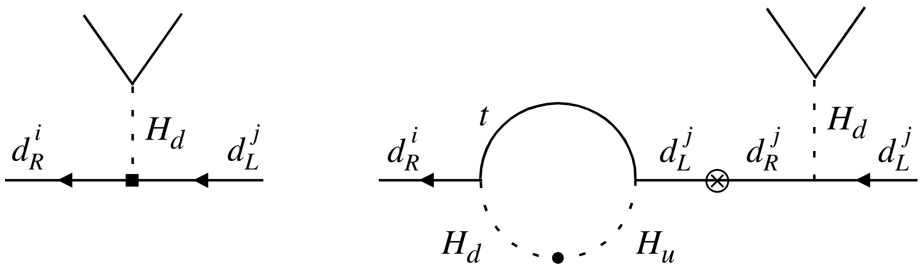
$$\mathcal{H}^{\rm eff}_{\Delta F=1} = \mathcal{H}^{\rm eff}_{\rm SM} + C_S O_S + C_P O_P + C'_S O'_S + C'_P O'_P \ + \ \text{h.c.}\,, \tag{2.15}$$

where $\mathcal{H}^{\rm eff}_{\rm SM}$ denotes the SM basis of $\Delta F = 1$ operators (see e.g. [5]) and

$$\begin{aligned} O_S &= \overline{d}^i_R d^j_L \overline{\ell}\ell\,, \qquad & O_P &= \overline{d}^i_R d^j_L \overline{\ell}\gamma_5\ell\,, \\ O'_S &= \overline{d}^i_L d^j_R \overline{\ell}\ell\,, \qquad & O'_P &= \overline{d}^i_L d^j_R \overline{\ell}\gamma_5\ell\,. \end{aligned} \tag{2.16}$$

One-loop contributions to $b \to s, d$ transitions in the non-supersymmetric 2HDM have been discussed in detail in refs. [5, 9, 11]. This analysis requires, apparently, the evaluation of a large number of box and penguin diagrams. However, the calculation can be strongly simplified by working in the limit $g = 0$, which has proved to be the most convenient framework to discuss leading $\tan\beta$ contributions to scalar FCNC amplitudes. In this limit it is easy to realize that box diagrams can be neglected, as long as we ignore terms suppressed by two lepton Yukawa couplings.

Penguin (and self-energy) diagrams can be evaluated by means of the effective Yukawa interaction discussed before, as shown in figure 2a. The limitation of the latter is that only the leading dimension-four operator $H_u^* \overline{d}_R Q_L$ has been considered. Higher-dimensional operators with more powers of H_u, which contribute to



a) b)

Figure 2: Feynman diagrams relevant to the $d_R^i \to d_L^j \ell_L \ell_R$ amplitude: a) leading contribution computed by means of the effective Yukawa interaction; b) one-loop contribution to the mass mixing in the 2HDM.

the mass diagonalization when H_u acquires a v.e.v., have been neglected. The coefficients of these higher-dimensional operators are necessarily suppressed by the inverse power of the heavy scale of the theory, namely the charged Higgs mass in the non-supersymmetric 2HDM. Therefore, using $\mathcal{L}_{\rm FCNC}^{\rm eff}$, we can control only the leading term in an expansion in powers of v_u/M_{H^+}. This leads to the following initial conditions for the Wilson coefficients in (2.15):

$$
\begin{aligned}
C_S = C_P &= \frac{y_i^d y_\ell y_t^2 \bar\lambda_{ij}^t}{32\pi^2} \times \frac{1}{M_{H^+}^2}\left[\log\left(\frac{M_{H^+}^2}{m_t^2}\right) + \mathcal{O}\left(\frac{m_t^2}{M_{H^+}^2}\right)\right], \\
C_S' = -C_P' &= \frac{y_j^d y_\ell y_t^2 \bar\lambda_{ji}^t}{32\pi^2} \times \frac{1}{M_{H^+}^2}\left[\log\left(\frac{M_{H^+}^2}{m_t^2}\right) + \mathcal{O}\left(\frac{m_t^2}{M_{H^+}^2}\right)\right], \qquad (2.17)
\end{aligned}
$$

where subleading $\mathcal{O}(1/\tan\beta)$ terms have been neglected. These results are compatible with those in refs. [5, 9, 11] and, even though $\mathcal{O}(m_t^2/M_{H^+}^2)$ corrections are missing, eqs. (2.17) have the advantage of incorporating higher-order $\tan\beta$ terms, hidden in y_i^d and $\bar\lambda_{ij}^t$ [see eqs. (2.5) and (2.10)]. As long as we neglect terms suppressed by additional down-type Yukawa couplings and/or additional off-diagonal CKM factors, these are the only source of $\tan\beta$-enhanced corrections.

It is worthwhile to stress that the subleading $\mathcal{O}(m_t^2/M_{H^+}^2)$ terms in eqs. (2.17) can also be computed in a rather simple way. To this purpose we only need to perform a complete diagonalization of the off-diagonal mass term generated at one loop, which is equivalent to computing the diagram in figure 2b (in the gaugeless limit). As a result, we find that the term within square brackets in eqs. (2.17) should be replaced by

$$
\frac{M_{H^+}^2}{M_{H^+}^2 - m_t^2}\log\left(\frac{M_{H^+}^2}{m_t^2}\right), \qquad (2.18)
$$

in agreement with the results of refs. [5, 9, 11].

Full one-loop calculations in the supersymmetric case have been performed in ref. [4, 5], complementing the result of ref. [3] obtained by means of the effective Yukawa couplings. As expected, the latter provides an excellent approximation to the full result in the limit of heavy squark masses. The supersymmetric contribution to $C_S = C_P$ obtained by means of $\mathcal{L}^{\rm eff}_{\rm FCNC}$ can be written as

$$C_S = -\frac{y_i^d y_\ell y_t^2 \bar\lambda^t_{ij}}{32\pi^2} \times \frac{A}{BM^2_{\tilde t_L}\left[1+\epsilon_0\tan\beta\right]} f\left(x_{\mu L}, x_{RL}\right), \tag{2.19}$$

$$= -\frac{G_F^2}{4\pi^2} \times \frac{m_{d_i} m_\ell m_t^2 \bar\lambda^t_{ij}\tan^2\beta}{1+\tan\beta(\epsilon_0+\epsilon_Y y_t^2\delta_{i3})} \times \frac{\mu A\tan\beta}{M^2_{\tilde t_L} M_A^2[1+\epsilon_0\tan\beta]} f\left(x_{\mu L}, x_{RL}\right). \tag{2.20}$$

The comparison between (2.19) and (2.20) illustrates the origin of the various $\tan\beta$ factors. Some comments are in order:

1. The third $\tan\beta$ factor in the numerator is not directly related to the down-type Yukawa couplings. As anticipated, with a different choice of parameters it can be eliminated in favour of the ratio A/B.

2. Higher-order corrections enhanced by $\tan\beta$, shown explicitly in eq. (2.20), cannot be neglected (similarly to the $b\to s\gamma$ case [10]). For $\tan\beta \gtrsim 30$ these higher-order terms are numerically more important than the remaining one-loop corrections not described by $\mathcal{L}^{\rm eff}_{\rm FCNC}$. By means of eq. (2.20) we take into account all terms of the type $(G_F m_b m_\ell)\times(\bar\alpha/\pi)^{n+1}(\tan\beta)^{n+3}$, where $n\geq 0$ and $\bar\alpha$ denotes either α_S or $y_t^2/(4\pi)$.

3. The $\tan\beta$-enhanced corrections to the $\bar t_R b_L H^+$ vertex, which play an important role in $b\to s\gamma$ although formally subleading [10], are not relevant here owing to the strong overall suppression of the charged-Higgs contribution with respect to the chargino one.

We conclude this section with a generic comparison between scalar FCNC amplitudes at large $\tan\beta$, both in the 2HDM and in the MSSM, and ordinary SM vector-type amplitudes. On general grounds, the former are suppressed with respect to the latter by a factor

$$\frac{M_W^2}{M_A^2} \times \frac{m_{d_i} m_\ell}{m_t^2} \times \tan^n\beta, \tag{2.21}$$

where $n\approx 2$ in the non-supersymmetric case and $n\lesssim 3$ in the MSSM. An exception to this rule is provided by $P\to\ell^+\ell^-$ decays, discussed in detail in section 4; here the helicity suppression of the SM matrix element leads to replacing the factor $m_{d_i}m_\ell$ in (2.21) by $m^2_{d_i}$. Taking into account the approximate numerical relations

$$\frac{m_s}{m_t}, \frac{m_\mu}{m_t} \sim \mathcal{O}\left(\frac{1}{\tan^2\beta}\right) \qquad \text{and} \qquad \frac{m_b}{m_t}, \frac{m_\tau}{m_t} \sim \mathcal{O}\left(\frac{1}{\tan\beta}\right) \tag{2.22}$$

that hold for $30\lesssim\tan\beta\lesssim 50$, the general picture can be summarized as follows.

JHEP11(2001)001

$\underline{b \to (s,d)\tau^+\tau^-}$: non-standard scalar contributions could have a sizeable impact already at the level of inclusive transitions. The magnitude of the scalar amplitude could be substantially larger than the SM one in the MSSM, whereas it is typically smaller than the SM one in the non-supersymmetric case.

$\underline{b \to (s,d)\mu^+\mu^-}$: the impact of scalar currents is almost negligible in inclusive transitions (as explicitly shown in ref. [5]), but large effects are still possible in the exclusive dilepton decays, especially within the MSSM. Indeed the most stringent constraint on $\Delta F = 1$ scalar currents –under the hypothesis of minimal flavour violation– is at present derived from the experimental bound on $B(B_s \to \mu^+\mu^-)$ (see section 4).

$\underline{s \to d\mu^+\mu^-}$: scalar current contributions could reach at most a few percent of the SM short-distance amplitude in $K_L \to \mu^+\mu^-$. Given the theoretical uncertainties affecting long-distance contributions to this mode [12], these non-standard effects are not detectable. We stress, however, that this conclusion holds only under the hypothesis of minimal flavour violation. If the CKM matrix is not the only source of flavour mixing, the $\epsilon_{Y,0}$ parameters are not flavour-diagonal and the $H_d^0 \overline{d}_R^i d_L^j$ coupling is not necessarily proportional to λ^t_{ij}. Within this more general framework, it is then possible to overcome the strong λ^t_{12} suppression of $s \to d\mu^+\mu^-$ amplitudes and to generate $\mathcal{O}(1)$ effects also in K decays. A detailed analysis of this scenario is beyond the purpose of this paper, where we shall restrict our attention to the hypothesis of minimal flavour violation and, henceforth, to B physics.

3. $B_{d,s}$–$\bar{B}_{d,s}$ mixing

3.1 Generalities

Within new-physics scenarios where flavour mixing is governed only by the CKM matrix, the effective Hamiltonian relevant to $B_{d,s}$–$\bar{B}_{d,s}$ mixing can be written as

$$H_{\rm eff}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 \left[V_{tb}^* V_{t(d,s)}\right]^2 \sum_i C_i(\mu) Q_i \ + \ \text{h.c.}\,, \tag{3.1}$$

where the full basis of dimension-six operators can be found, for example in ref. [13]. In addition to the SM operator

$$Q^{VLL} = \bar{b}_L \gamma_\mu q_L \bar{b}_L \gamma^\mu q_L\,, \tag{3.2}$$

those relevant at large $\tan\beta$ are given by

$$\begin{aligned} Q^{SLL} &= \bar{b}_R q_L \bar{b}_R q_L\,, \\ Q^{SLR} &= \bar{b}_R q_L \bar{b}_L q_R\,, \\ Q^{VRR} &= \bar{b}_R \gamma_\mu q_R \bar{b}_R \gamma^\mu q_R\,, \end{aligned} \tag{3.3}$$

where $q = d, s$.

All the Wilson coefficients of the four operators in eqs. (3.2) and (3.3) receives one-particle irreducible one-loop contributions from box diagrams. However, as pointed out in ref. [6], in the case of Q^{SLR} a sizeable contribution at large $\tan\beta$ is also provided by the reducible two-loop diagram in figure 3.

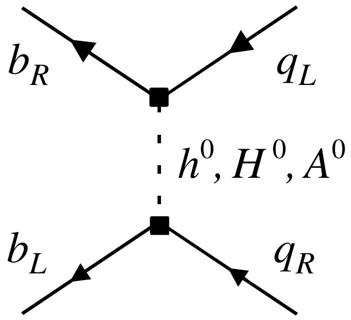


Figure 3: Double-penguin diagram contributing to C^{SLR}; the black boxes denote the effective FCNC Yukawa interaction of eq. (2.11).

The Wilson coefficients of the three operators in eqs. (3.3) are all suppressed by small down-type masses, corresponding to the right-handed fields, but are possibly enhanced by appropriate $\tan\beta$ factors, as we shall discuss in the following. The smallness of m_d implies that only Q^{SLL} plays a significant role in ΔM_d. On the other hand, both Q^{SLL} and Q^{SLR} are potentially relevant to ΔM_s. Despite a $\tan^4\beta$ enhancement, the contribution of Q^{VRR} is practically negligible for both ΔM_d and ΔM_s.

Following the notation of ref. [6], the $B_{d,s}$–$\bar{B}_{d,s}$ mass difference can be written as

$$\Delta M_{d,s} = \frac{G_F^2 M_W^2}{6\pi^2} \eta_B m_{B_{d,s}} (\hat{B}_{B_{d,s}} F_{B_{d,s}}^2) F_{tt}^{d,s} |V_{t(d,s)}|^2 \,, \tag{3.4}$$

where $(\hat{B}_{B_{d,s}} F_{B_{d,s}}^2)$ parametrize the hadronic matrix elements, to be determined through non-perturbative methods, and η_B is the QCD renormalization-group factor of the SM operator, given by $\eta_B = 0.55$ at next-to-leading order (NLO). The real functions $F_{tt}^{d,s}$, encoding short-distance contributions, are conveniently decomposed as

$$F_{tt}^{d,s} = S_0(x_{tW})[1 + f_{d,s}] \,, \tag{3.5}$$

where $x_{tW} = m_t^2/M_W^2$, $S_0(x_{tW})$ accounts for the SM box (see appendix) and all non-standard effects are included in $f_{d,s}$. Expressing $f_{d,s}$ in terms of the (non-standard) contributions to the Wilson coefficients of the four-fermion operators in eqs. (3.2) and (3.3), we can write

$$f_{d,s} = \frac{1}{4S_0(x_{tW})} \left[C^{VLL}(\mu_t) + C^{VRR}(\mu_t) + 4\bar{P}^{SLR} C^{SLR}(\mu_t) + 4\bar{P}^{SLL} C^{SLL}(\mu_t) \right] \,, \tag{3.6}$$

where $\mu_t = \mathcal{O}(m_t)$ and the explicit expressions of the $\bar{P}^i$, taking into account renormalization-group QCD corrections [13] and matrix elements of the scalar operators — normalized to the SM one — can be found in ref. [6].[1] Assuming B-parameters

[1] In principle the QCD correction factor of non-standard vector operators is not exactly η_B; however, this difference can be ignored at the level of accuracy we are interested in. Similarly, we will neglect all finite QCD corrections to the initial conditions of the non-standard Wilson coefficients and corrections due to the running between μ_t and the new-physics scale.

of the scalar operators equal to one, setting $m_b(\mu_b) = 4.2$ GeV, $\alpha_s(M_Z) = 0.118$ and $\hat{B}_{B_{d,s}} = 1.3 \pm 0.2$ we find

$$4\bar{P}^{SLR} = 3.5 \pm 0.5\,, \qquad 4\bar{P}^{SLL} = -2.1 \pm 0.3\,. \tag{3.7}$$

As can be noted, QCD effects enhance the contribution of scalar operators, especially Q^{SLR}, thus even small new physics contributions to their Wilson coefficients may be relevant to the phenomenological analysis.

3.2 $\Delta B = 2$ box diagrams

Computing explicitly non-supersymmetric box diagrams with the exchange of $W^\pm$ and $H^\pm$ or two $H^\pm$ (Goldstone bosons included), in the large-$\tan\beta$ limit, we obtain the following initial conditions for the Wilson coefficients

$$\begin{aligned}
C_H^{VLL} &= \frac{2}{\tan^2\beta}\left[L_2(x_{tW}, x_{tW}, x_{HW}) - 4L_1(x_{tW}, x_{tW}, x_{HW})\right],\\
C_H^{VRR} &= \frac{m_b^2 m_{d,s}^2 \tan^4\beta}{M_W^2 M_{H^\pm}^2}\left[L_3(x_{tH}, x_{tH}, 1) - 2L_3(x_{tH}, 0, 1) + L_3(0,0,1)\right],\\
C_H^{SLL} &= \frac{4m_b^2}{M_W^2}\left[L_1(x_{tW}, x_{tW}, 1) + L_1(x_{tH}, x_{tH}, 1) - 2L_1(x_{tH}, x_{tH}, x_{WH})\right],\\
C_H^{SLR} &= \frac{8m_b m_{d,s}\tan^2\beta}{M_{H^\pm}^2}\left[-L_3(x_{tH}, x_{tH}, x_{WH}) + 2L_3(x_{tH}, 0, x_{WH})\right.\\
&\qquad\left. -L_3(0,0,x_{WH}) + x_{HW}L_1(x_{tH}, x_{tH}, x_{WH})\right],
\end{aligned} \tag{3.8}$$

where, as usual, $x_{ab} = M_a^2/M_b^2$ and the loop functions $L_i(x, y, z)$ are defined in the appendix. These results are in agreement with those recently reported in ref. [6].

The charged-Higgs contribution to C_H^{VLL} is directly suppressed by two inverse powers of $\tan\beta$ and turns out to be completely negligible. The strong $\tan\beta$ enhancement of C_H^{VRR} is more than compensated by the down-type quark-mass terms: employing the power counting in eq. (2.22), C_H^{VRR} is effectively of $\mathcal{O}(1/\tan^2\beta)$ at most, and indeed it is numerically irrelevant. Similarly, also C_H^{SLL} is negligible being effectively of $\mathcal{O}(1/\tan^2\beta)$. The potentially largest contribution — still suppressed with respect to the SM one — is provided by C_H^{SLR}, which in the B_s–$\bar{B}_s$ case is effectively suppressed by only one inverse power of $\tan\beta$. tFor $M_H \approx m_t$, C_H^{SLR} can induce at most a 10% correction to F_{tt}^s:

$$\begin{aligned}
\delta f_s^{\rm 2HDM-box} &= \frac{\bar{P}_2^{SLR} C_H^{SLR}}{S_0(x_{tW})} = \frac{m_b m_s \tan^2\beta}{M_W^2}\left[-1.5 - (x_{tH}-1) + \mathcal{O}(x_{tH}-1)^2\right]\\
&\approx \left[-0.10 - 0.07(x_{tH}-1) + \mathcal{O}(x_{tH}-1)^2\right]\left(\frac{\tan\beta}{50}\right)^2
\end{aligned} \tag{3.9}$$

[$m_b(\mu_t) \approx 3$ GeV, $m_s(\mu_t) \approx 60$ MeV]. We can therefore conclude that non-supersymmetric 2HDM box contributions at large $\tan\beta$ do not induce appreciable effects in B^0–$\bar{B}^0$ mixing.

An important point to note is the following: whereas each down-type mass term is compensated by a corresponding $\tan\beta$ factor, as naively expected, in C_H^{SLR} and C_H^{VRR}, this does not occur in C_H^{VLL} and C_H^{SLL}. In the case of C_H^{SLL}, the reason of the additional $1/\tan^2\beta$ suppression is its vanishing in the absence of $\mathrm{U}(1)_d$ breaking. Similarly to the case of $\Delta B = 1$ amplitudes, this fact can easily be understood in the gaugeless limit. As shown in figure 4: the two H_u–H_d mixing terms compensate the two $\tan\beta$ factors of the Yukawa couplings and, as a result, C_H^{SLL} has no explicit $\tan\beta$ dependence. As shown in section 2, $\mathrm{U}(1)_d$ breaking is not necessarily suppressed at large $\tan\beta$ in the supersymmetric case, when the H_u–H_d mixing is replaced by the $\widetilde{H}_u$–$\widetilde{H}_d$ one. We can therefore expect a $\tan^2\beta$-enhanced chargino-squark contribution to C^{SLL}.

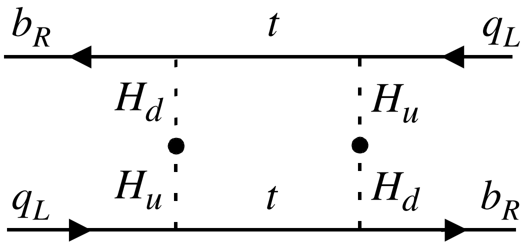


Figure 4: Leading box-diagram contribution to C^{SLL} in the non-supersymmetric 2HDM.

The full one-loop contributions to C^{VLL} and C^{SLL} generated by chargino-squark diagrams in the MSSM can be written as

$$C_\chi^{VLL} = 4[f(\tilde{u},\tilde{u}) - 2f(\tilde{u},\tilde{t}) + f(\tilde{t},\tilde{t})]\,, \tag{3.10}$$

$$C_\chi^{SLL} = 4[g(\tilde{t},\tilde{t}) - 2g(\tilde{u},\tilde{t}) + g(\tilde{t},\tilde{t})]\,, \tag{3.11}$$

where

$$\begin{aligned} f(\tilde{u},\tilde{t}) &= \sum_{i,j,h,k=1,2} x_{W\chi_j} Y_{i\tilde{u}_h} Y_{i\tilde{t}_k} Y_{j\tilde{t}_k} Y_{j\tilde{u}_h} L_3(x_{\tilde{t}_k\chi_j}, x_{\tilde{u}_h\chi_j}, x_{\chi_i\chi_j})\,, \\ g(\tilde{u},\tilde{t}) &= 4 \sum_{i,j,h,k=1,2} x_{W\chi_j} \frac{M_{\chi_i}}{M_{\chi_j}} Y_{i\tilde{u}_h} Z^b_{i\tilde{t}_k} Y_{j\tilde{t}_k} Z^b_{j\tilde{u}_h} L_4(x_{\tilde{t}_k\chi_j}, x_{\tilde{u}_h\chi_j}, x_{\chi_i\chi_j})\,, \end{aligned} \tag{3.12}$$

and $Y_{j\tilde{u}_h}$ and $Z^q_{j\tilde{u}_h}$ are defined as in ref. [14].[2]

The chargino-squark contribution to the Wilson coefficient of the standard vector operator (C_χ^{VLL}), which includes the supersymmetrization of the ordinary W-box diagram, has no special features at large $\tan\beta$: it is not suppressed or enhanced. For this reason, and also because C_χ^{VLL} has been extensively discussed in the literature (see, in particular, refs. [14, 15]), we will not discuss it further.

Contrary to C_χ^{VLL}, the chargino-squark contribution to the scalar operator Q^{SLL} has not been discussed before and, as anticipated, it has an interesting behaviour at large $\tan\beta$. In order to provide a compact analytical expression and a numerical evaluation of C_χ^{SLL}, we employ the following simplifying assumptions:

[2] The superscript in $Z^q_{j\tilde{u}_h}$ refers to the down-type Yukawa coupling involved.

- we neglect up and charm Yukawa couplings and any non-degeneracy among the first two generations of squarks: $M_{\tilde{q}_i} = \tilde{M}$ for $q = u, c$ and $i = 1, 2$ [as already implicitly assumed in eqs. (3.10)–(3.12)];

- we neglect off-diagonal terms in the chargino mass matrix (or we assume $M_W \ll \max\{M_2, \mu\}$), so that $M_{\chi_1} \approx M_2$ and $M_{\chi_2} \approx \mu$;

- we neglect the left-right mixing of the squarks, but in the stop sector; there we introduce a mixing angle ($-\pi/2 < \theta_{\tilde{t}} < \pi/2$) satisfying the relation

$$\sin 2\theta_{\tilde{t}} = \frac{2m_t(A - \mu\cot\beta)}{M^2_{\tilde{t}_1} - M^2_{\tilde{t}_2}} \approx \frac{2m_t A}{M^2_{\tilde{t}_1} - M^2_{\tilde{t}_2}}\,, \tag{3.13}$$

  where the $\tilde{t}_1$ mass eigenstate ($M_{\tilde{t}_1} < M_{\tilde{t}_2}$) can be identified with the right-handed stop for small $\theta_{\tilde{t}}$.

In the limit where $M_{\tilde{t}_2}$ is degenerate with the first two generations of squarks, while $M_{\tilde{t}_1}$ is kept lighter, we obtain the following rather simple expression:

$$g(\tilde{t},\tilde{t}) = \frac{y_t^2 y_b^2 \sin^2 2\theta_t}{32 G_F^2 M_W^2 \mu^2}\, G(x_{\tilde{t}_1\mu}, x_{\tilde{t}_2\mu}) \qquad \left[g(\tilde{u},\tilde{t}) = g(\tilde{u},\tilde{u}) = 0\right]\ , \tag{3.14}$$

where

$$\begin{aligned} G(x,y) &= L_4(x,x,1) - 2L_4(x,y,1) + L_4(y,y,1) \\ &= \frac{G_2(y)}{y^2}(x-y)^2 + \mathcal{O}(x-y)^3\,, \end{aligned} \tag{3.15}$$

and $G_2(y)$ can be found in the appendix. The expansion of $G(x,y)$ around $x \approx y$ lets us further simplify eq. (3.14), eliminating $\sin 2\theta_{\tilde{t}}$ in favour of A. We then obtain

$$g(\tilde{t},\tilde{t}) = \frac{m_t^4 m_b^2 \tan^2\beta}{M_W^2 \mu^4 \left[1 + \tan\beta(\epsilon_0 + \epsilon_Y y_t^2)\right]^2} \left(\frac{\mu A}{M^2_{\tilde{t}_L}}\right)^2 G_2(x_{L\mu})\left[1 + \mathcal{O}(1 - x_{LR})\right]\ , \tag{3.16}$$

where the Yukawa couplings have been expressed in terms of quark masses. Note the analogy of eq. (3.16) to eq. (2.14): in both cases the overall effect is ruled by the ratio $\mu A/M^2_{\tilde{t}_L}$, which appears squared in the $\Delta B = 2$ amplitude. However, since the effective FCNC interaction of eq. (2.9) does not enter the box amplitude, the $\tan\beta$ enhancement of the latter simply scale with the power of down-type Yukawa couplings. As we shall discuss later on, for large $\tan\beta$, small μ ($\mu \lesssim 200$ GeV) and $\mu A/M^2_{\tilde{t}_L} = \mathcal{O}(1)$, this supersymmetric scalar contribution has a small but non-negligible impact on ΔM_d and ΔM_s.

3.3 Double-penguin diagrams

Double-penguin diagrams are formally a higher-order (two-loop) effect; however, the large-$\tan\beta$ growth of the effective Yukawa coupling make them numerically competing with one-loop contributions in the B_s–$\bar{B}_s$ case [6].

The structure of eq. (2.11) leads, in general, to a vanishing result when the Higgs fields are contracted to generate effective operators of the type $\bar{b}_R q_L \bar{b}_R q_L$ or $\bar{b}_L q_R \bar{b}_L q_R$ $(q = s, d)$. On the other hand, a non-vanishing result is obtained for effective operators of the type $\bar{b}_R q_L \bar{b}_L q_R$, or when the combination of Higgs fields explicitly shown in eq. (2.11) is contracted with its Hermitian conjugate. As a result, only C^{SLR} is affected by double-penguin contributions.

In general, the double-penguin contribution to C^{SLR} at large $\tan\beta$ can be written as

$$C^{SLR} = -\frac{8\pi^2 y_b y_{s,d}}{G_F^2 M_W^2} \mathcal{F}_+ \left(\frac{\epsilon_Y y_t^2 \tan\beta}{1+\epsilon_0 \tan\beta} \right)^2 , \tag{3.17}$$

where [6]

$$\mathcal{F}_+ = \frac{\cos^2(\alpha-\beta)}{M_{h^0}^2} + \frac{\sin^2(\alpha-\beta)}{M_{H^0}^2} + \frac{1}{M_{A^0}^2} \stackrel{M_A^2 \gg M_W^2}{\longrightarrow} \frac{2}{M_A^2} . \tag{3.18}$$

Within the non-supersymmetric case, the $1/\tan\beta$ suppression of $\epsilon_Y^{\rm 2HDM}$ leads to negligible effects for both ΔM_d and ΔM_s. On the contrary, the supersymmetric contribution to ΔM_s can compete with the SM one. Expressing Yukawa couplings in terms of quark masses, we find

$$C_\chi^{SLR} = -\frac{G_F m_b m_s m_t^4}{\sqrt{2}\pi^2 M_W^2} \frac{\tan^4\beta \mathcal{F}_+ f^2(x_{\mu L}, x_{RL})}{(1+\epsilon_0\tan\beta)^3[1+\tan\beta(\epsilon_0+\epsilon_Y y_t^2)]} \left(\frac{\mu A}{M_{\tilde{t}_L}^2} \right)^2 . \tag{3.19}$$

The $\tan^4\beta$ enhancement of this coefficient is partially compensated by the m_s suppression and the additional $1/16\pi^2$ (due to the two-loop order). As a result, it is of the same order as the chargino-stop box contribution to C_χ^{SLL}. Neglecting higher-order $\tan\beta$ terms one finds

$$\begin{aligned} \frac{C_\chi^{SLL}}{C_\chi^{SLR}} &\approx \frac{4\pi^2}{5\sqrt{2}} \times \frac{m_b M_A^2 h(x_{\mu L}, x_{RL})}{m_s \mu^4 G_F \tan^2\beta} \\ &\approx \left(\frac{M_A}{\mu}\right)^2 \left(\frac{100\ \mathrm{GeV}}{\mu}\right)^2 \left(\frac{50}{\tan\beta}\right)^2 h(x_{\mu L}, x_{RL}) , \end{aligned} \tag{3.20}$$

where the function $h(x_{\mu L}, x_{RL})$, normalized to $h(1,1) = 1$, is plotted in figure 5.

3.4 Numerical bounds from $\Delta M_{d,s}$ in the MSSM

In the case of ΔM_d, the only relevant $\Delta B = 2$ scalar operator is Q^{SLL}. Neglecting possible new-physics effects in the standard vector-current operator, according to

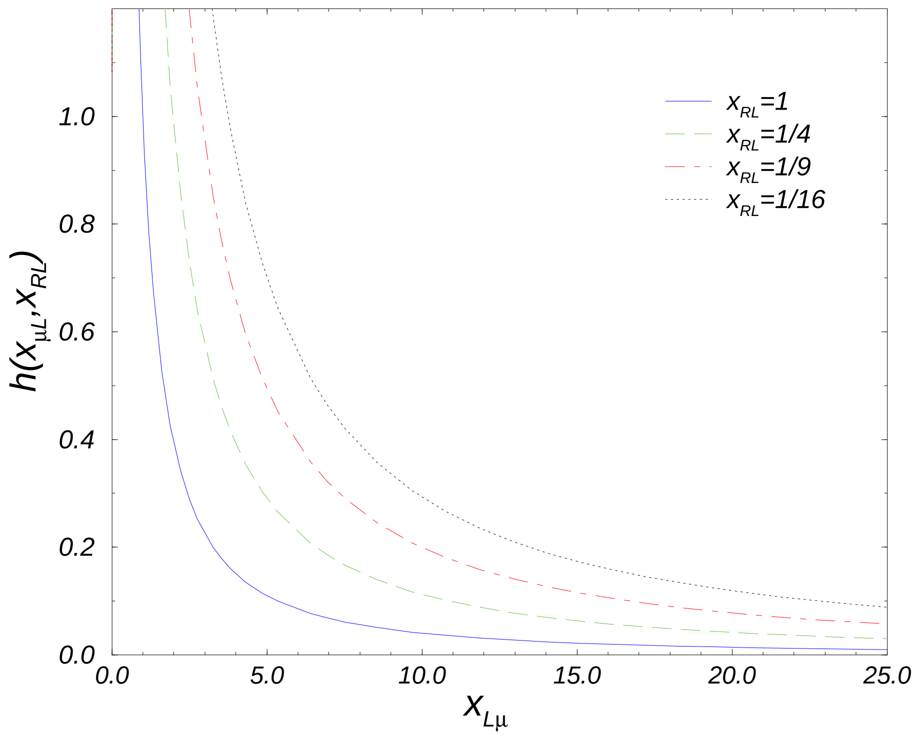


Figure 5: $h(x_{\mu L}, x_{RL})$ as a function of $x_{L\mu}$ $(= 1/x_{\mu L})$ for different values of x_{RL}.

eqs. (3.14) and (3.6) the large-$\tan\beta$ contribution to f_d is given by

$$f_d = \frac{\bar{P}^{SLL} C^{SLL}(\mu_t)}{S_0(x_{tW})} \approx \frac{4\bar{P}^{SLL}}{S_0(x_{tW})} \frac{m_t^4 m_b^2 \tan^2\beta}{M_W^2 \mu^4} \left(\frac{\mu A}{M_{\tilde{t}_L}^2}\right)^2 \frac{G(x_{R\mu}, x_{L\mu})}{(1-x_{RL})^2} \,. \tag{3.21}$$

Imposing that f_d varies within the range allowed by the measurement of ΔM_d, we can derive a set of bounds on the quantity $\mu A/M_{\tilde{t}_L}^2$ for different values of stop masses ($M_{\tilde{t}_L}$ and $M_{\tilde{t}_R}$) and μ, as shown in figure 6. Although the experimental determination of ΔM_d is rather precise, once the uncertainties on $\hat{B}_{B_d} F_{B_d}^2$ and $|V_{td}|$ are taken into account, f_d can reach values close to 1, which we take as a reference figure. As can be noted from figure 6, the bounds on $\mu A/m_{\tilde{t}_L}^2$ thus obtained are not severe. Even for small μ ($\mu \approx 100$ GeV), light $m_{\tilde{t}_R}$ ($m_{\tilde{t}_R} \approx 200$ GeV) and a large stop splitting, the upper bound on $\mu A/m_{\tilde{t}_L}^2$ is above unity, condition that is naturally satisfied within our scenario and that still allows large effects in $\Delta B = 1$ processes.

Constraints even less strict on $\mu A/M_{\tilde{t}_L}^2$ can in general be set by using the present experimental lower bound for the B_s–$\bar{B}_s$ mixing. In this case also the double-penguin contribution to Q^{SLR} has to be taken into account. As shown in eq. (3.20), for light μ and $M_A \sim \mu$, the large $\tan\beta$ contributions to C^{SLR} and C^{SLL} are comparable and tend to cancel out in f_s, owing to the opposite sign between $\bar{P}^{SLL}$ and $\bar{P}^{SLR}$. Only if M_A is light and $\mu \gg M_A$, or when the double-penguin contribution completely dominates, does one obtain a non-trivial bound.

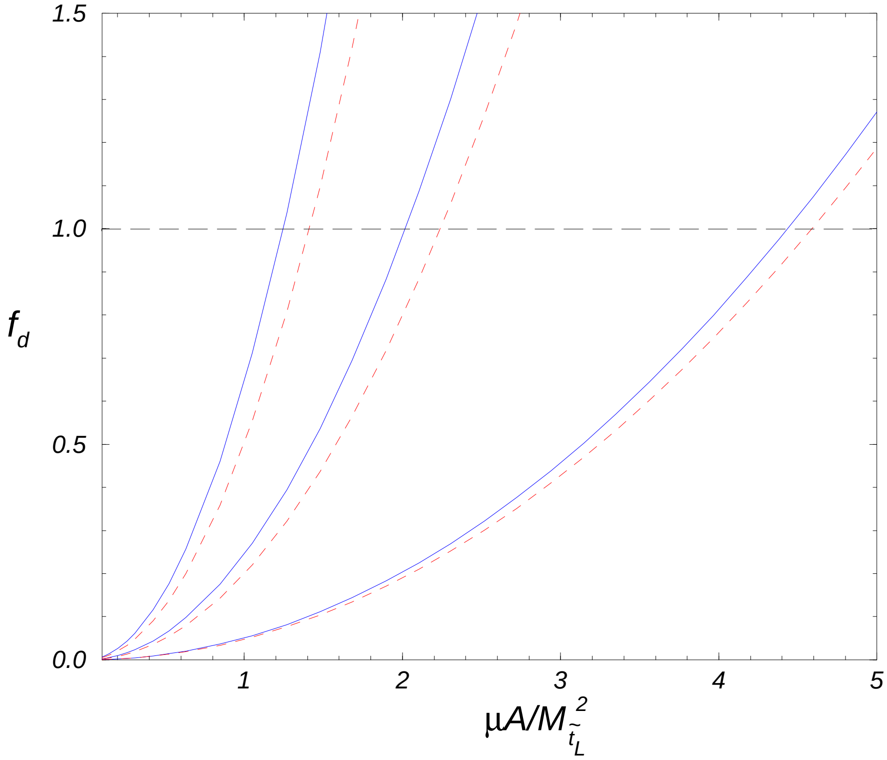


Figure 6: f_d as a function of $\mu A/M^2_{\tilde{t}_L}$ for $\mu = 100$ GeV and different stop masses. The three solid (dashed) curves from the left are obtained for $\{M_{\tilde{t}_R}(\text{GeV}), M_{\tilde{t}_L}(\text{GeV})\} = \{200,\ 500\ (300)\}, \{300,\ 1000\ (500)\}, \{500,\ 1000\ (800)\}$.

In the limit where the double-penguin contribution dominates, higher-order $\tan\beta$-enhanced corrections are numerically relevant and cannot be ignored. In particular, terms proportional to [7]

$$\epsilon_0 = \frac{2\alpha_S}{3\pi} \frac{\mu M_3}{M^2_{\tilde{d}_L}} f\left(x_{3L}, x_{RL}\right), \tag{3.22}$$

turn out to be very large. In order to obtain a simple expression, we can neglect the contribution of $\epsilon_Y y_t^2$ with respect to ϵ_0 in the denominator of eq. (3.19), and set $\epsilon_0 \approx 1/100$, as obtained in the case of degenerate sparticles. We also set $\epsilon_0 > 0$, following the indication $\mu > 0$ from $b \to s\gamma$ and $(g-2)_\mu$ [16]. We then obtain

$$f_s = \frac{\bar{P}^{SLR} C^{SLR}(\mu_t)}{S_0(x_{tW})} \approx -0.4 \left[\frac{3(\tan\beta/50)}{2 + (\tan\beta/50)}\right]^4 \left(\frac{200\ \text{GeV}}{M_A}\right)^2 \left(\frac{\mu A f(x_{\mu L}, x_{RL})}{M^2_{\tilde{t}_L}}\right)^2 . \tag{3.23}$$

Since the present lower bound on ΔM_s implies a lower bound on f_s of about -0.5 [6], we conclude that also in this scenario values of $\mu A/M^2_{\tilde{t}_L} \lesssim 1$ are still allowed. Note that a much stronger bound would have been obtained without the inclusion of higher-order terms: for $\tan\beta \approx 50$ these reduce the lowest-order result almost by a factor 5.

4. Phenomenology of $B_{s,d} \to \ell^+\ell^-$ decays

4.1 Effective hamiltonian

Within the SM there is only one dimension-six operator generating a non-vanishing contribution to $B_q \to \ell^+\ell^-$ decays, namely

$$Q_{10} = \bar{b}_L \gamma^\mu q_L \bar{\ell} \gamma_\mu \gamma_5 \ell\,. \tag{4.1}$$

Being scale-invariant and completely dominated by short-distance dynamics, its Wilson coefficient is well approximated by the lowest-order result [17]:

$$C_{10}^{\rm SM}\big|_{\rm LO} = \frac{G_F^2 M_W^2 V_{tq}^*}{\pi^2}\, Y_0(x_{tW}) \approx \frac{G_F^2 M_W^2 V_{tq}^*}{\pi^2} \left(\frac{m_t(\mu_t)}{166 \text{ GeV}}\right)^{1.56}\,. \tag{4.2}$$

The full NLO expression of $C_{10}^{\rm SM}$ can be found in [18].

The additional scalar interactions that appear in the SM extensions under investigation are fully described by the operators in eq. (2.16). If the lepton Yukawa coupling is not negligible, as in the case of a $\tau^+\tau^-$ final state, in principle we should take into account also the right-handed vector-current operator Q_{10}', obtained by Q_{10} under the exchange $(q,b)_L \to (q,b)_R$. However, as can be expected from a naive extrapolation of the results in the previous section, the numerical impact of this operator is always negligible. Within the 2HDM one finds

$$C_{10}^{\prime H} = \frac{G_F^2 M_W^2 V_{tq}^*}{\pi^2} \frac{m_q m_b m_\ell^2}{4 M_W^2 M_H^2} \tan^4\beta B_0(x_{tH})\,, \tag{4.3}$$

where $B_0(x) = [L_3(x,0,1) - L_3(0,0,1)]/4$. In the most favourable case, namely the $B_s \to \tau^+\tau^-$ decay, $C_{10}^{\prime H}$ leads at most to $\mathcal{O}(1\%)$ corrections to the SM result. A similar suppression is found also for the MSSM contribution to C_{10}'.

For completeness, we report here also the MSSM chargino-box contribution to $C_{S,P}$, which was not explicitly shown in section 2. This is given by

$$\begin{aligned} C_{S,P}^{\chi} = \frac{G_F^2 M_W^2}{2\pi^2} V_{3i}^* V_{3j} \sum_{h,k,m,n} & x_{W\chi_n} [Z_{m\tilde{t}_h}^b Z_{m\tilde{\nu}_k}^\ell Y_{n\tilde{\nu}_k} Y_{n\tilde{t}_h} L_3(x_{\tilde{\nu}_k\chi_n}, x_{\tilde{t}_h\chi_n}, x_{\chi_m\chi_n}) \pm \\ & \pm \frac{m_{\chi_m}}{m_{\chi_n}} Y_{m\tilde{t}_h} Z_{m\tilde{\nu}_k}^\ell Y_{n\tilde{\nu}_k} Z_{n\tilde{t}_h}^b L_4(x_{\tilde{\nu}_k\chi_n}, x_{\tilde{t}_h\chi_n}, x_{\chi_m\chi_n})] - (\tilde{t} \to \tilde{u})\,, \end{aligned} \tag{4.4}$$

where we have assumed diagonal chargino-lepton-slepton couplings, and terms suppressed by $m_{s,d}/m_b$ or by the mass difference between the first two generations of squarks have been neglected (under these hypotheses eq. (4.4) is agreement with the results of ref. [5]). Following the simplifying assumptions made for the chargino-squark contributions to B^0–$\bar{B}^0$ mixing, the above result becomes

$$\begin{aligned} C_S^\chi = C_P^\chi = \frac{G_F^2 M_W^2}{4\pi^2} V_{3i}^* V_{3j} \frac{m_b m_\ell \tan^2\beta}{\mu^2} & \left[\cos^2\theta_t L_3(x_{\tilde{\nu}\mu}, x_{\tilde{t}_1\mu}, x_{\tilde{W}\mu}) + \right. \\ & \left. + \sin^2\theta_t L_3(x_{\tilde{\nu}\mu}, x_{\tilde{t}_2\mu}, x_{\tilde{W}\mu}) - L_3(x_{\tilde{\nu}\mu}, x_{\tilde{u}\mu}, x_{\tilde{W}\mu})\right]. \end{aligned} \tag{4.5}$$

From this expression it is straightforward to check that, for large $\tan\beta$, box contributions cannot compete with the neutral Higgs-penguin coefficients in eq. (2.20), as already anticipated in section 2. We shall therefore neglect such terms in the following phenomenological discussion.

4.2 Branching ratios

Writing hadronic matrix elements of axial and pseudoscalar currents as

$$\langle \bar{0}|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}_q(p)\rangle = i p_\mu f_{B_q} \ , \qquad \langle \bar{0}|\bar{q}\gamma_5 b|\bar{B}_q(p)\rangle = -i f_{B_q}\frac{M^2_{B_q}}{(m_b+m_q)} \ , \tag{4.6}$$

the most general expression of $B_q \to \ell^+\ell^-$ branching ratios reads, according to our normalization of the effective operators:

$$\mathcal{B}\left(B_q \to \ell^+\ell^-\right) = \frac{f^2_{B_q} M_{B_q}\tau_{B_q} m^2_\ell}{8\pi}\left(1-\frac{4m^2_\ell}{M^2_{B_q}}\right)^{1/2}\left[\left(1-\frac{4m^2_\ell}{M^2_{B_q}}\right)\left|\frac{M^2_{B_q}(C_S-C'_S)}{2m_\ell(m_b+m_q)}\right|^2 + \left|\frac{M^2_{B_q}(C_P-C'_P)}{2m_\ell(m_b+m_q)}+C_{10}-C'_{10}\right|^2\right] , \tag{4.7}$$

where light-quark masses and Wilson coefficients are understood to be evaluated at a scale of $\mathcal{O}(m_b)$. Since the ratios $(C_{S,P}-C'_{S,P})/m_\ell$ are independent of m_ℓ, to a first approximation the relative weight of the various contributions is independent of m_ℓ, whereas the overall branching ratio scales like m^2_ℓ. This leads to a strong enhancement factor of the $\tau^+\tau^-$ modes that partially compensate their difficult detection.

Within the SM, employing the full NLO expression of $C_{10}^{\rm SM}$ [18], the branching ratios of the two B_s modes are given by

$$\mathcal{B}(B_s\to\mu^+\mu^-)\big|_{\rm SM} = 3.1\times 10^{-9}\left(\frac{f_{B_s}}{0.21\ \rm GeV}\right)^2\left(\frac{|V_{ts}|}{0.04}\right)^2\left(\frac{\tau_{B_s}}{1.6\ \rm ps}\right)\left(\frac{m_t(m_t)}{166\ \rm GeV}\right)^{3.12} \tag{4.8}$$

$$\mathcal{B}(B_s\to\tau^+\tau^-)\big|_{\rm SM} = 215\times\mathcal{B}(B_s\to\mu^+\mu^-)\big|_{\rm SM} \ , \tag{4.9}$$

whereas the corresponding B_d modes are both suppressed by an additional factor $|V_{td}/V_{ts}|^2 = (4.0\pm 0.8)\times 10^{-2}$. Note that the $\mathcal{B}\left(B_s\to\tau^+\tau^-\right)/\mathcal{B}\left(B_s\to\mu^+\mu^-\right)$ ratio in eq. (4.9) can be considered as a model-independent upper bound: if $C_{S,P}\propto m_\ell$ this ratio can only be smaller than in the SM case. At present the experimental bound closest to SM expectations is given by

$$\mathcal{B}\left(B_s\to\mu^+\mu^-\right) < 2.6\times 10^{-6}\ \ (95\%\ \ \rm CL)\ \ [19] \, . \tag{4.10}$$

Working in the limit where the supersymmetric contribution to $C_{S,P}$ in eq. (2.20) dominates over all other contributions, and neglecting terms suppressed by m_s/m_b

and m_μ/m_b, we find

$$\mathcal{B}(B_s \to \mu^+\mu^-)\big|_{\rm MSSM} = \frac{f_{B_s}^2 G_F^4 M_{B_s}^5 |V_{ts}|^2 m_\mu^2 \tau_{B_s}}{256\pi^5} \left(\frac{m_t(\mu_t)}{M_A}\right)^4 \left(\frac{\mu A}{M_{\tilde{t}_L}^2}\right)^2 \times$$
$$\times \left(\frac{\tan^3\beta\ f(x_{\mu L}, x_{RL})}{(1+\epsilon_0\tan\beta)[1+\tan\beta(\epsilon_0+\epsilon_Y y_t^2)]}\right)^2 . \quad (4.11)$$

Setting $\epsilon_0 \approx 1/100$ and neglecting the $\epsilon_Y y_t^2$ term in the denominator [as done in eq. (3.23)], and employing the reference values of f_{B_s}, τ_{B_s}, m_t and V_{ts} as in eq. (4.8), we obtain the following compact phenomenological expression

$$\mathcal{B}(B_s \to \mu^+\mu^-)\big|_{\rm MSSM} \approx 3\times 10^{-6} \frac{r^6}{\left(\frac{2}{3}+\frac{1}{3}r\right)^4} \left(\frac{200\ \mathrm{GeV}}{M_A}\right)^4 \left(\frac{\mu A f(x_{\mu L}, x_{RL})}{M_{\tilde{t}_L}^2}\right)^2 , \quad (4.12)$$

where $r = \tan\beta/50$. Given the weak constraints on $\mu A/M_{\tilde{t}_L}^2$ discussed in section 3, we cannot exclude a scenario where $\mathcal{B}(B_s \to \mu^+\mu^-)$ is just below its present experimental bound, as already pointed out in refs. [3, 4, 5]. However, we stress that this can occur only for rather extreme values of the parameter space, in particular for $\tan\beta \sim 50$, when higher-order $\tan\beta$-enhanced corrections are very large. For $\tan\beta \le 30$ it is already unlikely to find $\mathcal{B}(B_s \to \mu^+\mu^-)$ above the 10^{-7} level and for $\tan\beta \le 10$ supersymmetric contributions do not exceed the level of the SM ones. Concerning higher-order $\tan\beta$-enhanced terms, we recall that these are fully taken into account by means of eq. (4.11), once the appropriate expressions of ϵ_0 and ϵ_Y are used, whereas the approximate result in eq. (4.11) is valid only for $M_3 \sim M_{\tilde{q}} \sim \mu$ ($\mu > 0$).

In the scenario where the present discrepancy between experimental data and SM predictions on the anomalous magnetic moment of the muon [20, 21] is due to supersymmetric effects, the possibility of a sizeable enhancement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ certainly becomes more likely. Indeed in both cases supersymmetric contributions grow with $\tan\beta$. As recently pointed out in ref. [22], the correlation between these two observables becomes very strong in the constrained minimal supergravity scenario. Unfortunately, the situation is not so clear in the general framework of the MSSM. A useful tool to illustrate basic features of the supersymmetric contribution to $a_\mu = (g-2)_\mu/2$ is the expression

$$\frac{a_\mu^{\rm MSSM}}{15\times 10^{-10}} \approx 1.7 \left(\frac{\tan\beta}{50}\right) \left(\frac{500\ \mathrm{GeV}}{M_{\tilde{\chi}}}\right)^2 \left(\frac{M_{\tilde{\nu}}}{M_{\tilde{\chi}}}\right), \quad (4.13)$$

which provides a good approximation to the full one-loop result [16] in the limit of almost degenerate higgsinos and electroweak gauginos ($M_1 \sim M_2 \sim \mu \gg M_W$), and allowing a moderate splitting between slepton and chargino masses (the normalization of the l.h.s. is the present experimental error [20]). Comparing eqs. (4.12) and

(4.13) helps us draw the following general conclusions, whose validity goes beyond the approximations made to derive the two equations:

- The mild $\tan\beta$ dependence of $a_\mu^{\rm MSSM}$ allows significant supersymmetric contributions to the latter, even for $\tan\beta \leq 10$. In this case the effects on $\mathcal{B}(B_s \to \mu^+\mu^-)$ would be undetectable, at least before the LHC.
- $\mathcal{B}(B_s \to \mu^+\mu^-)^{\rm MSSM}$ strongly depends on A and M_A, which play almost no role in $a_\mu^{\rm MSSM}$. As a result, $\mathcal{B}(B_s \to \mu^+\mu^-)^{\rm MSSM}$ can be made arbitrarily small even if $a_\mu^{\rm MSSM}$ saturates the experimental result and $\tan\beta \sim 50$.
- A detection of $\mathcal{B}(B_s \to \mu^+\mu^-)$ at the 10^{-7} level (or above) would provide a strong support in favour of the supersymmetric explanation of the $(g-2)_\mu$ puzzle. This result could be interpreted in the MSSM framework only for $\tan\beta \gtrsim 30$ and light M_A, a scenario where it is difficult to keep $a_\mu^{\rm MSSM}$ below 10^{-9} unless chargino masses are unnaturally high.

Concerning $B_{s,d} \to \tau^+\tau^-$ modes, the following relations hold in the limit where scalar-current contributions are dominant:

$$\frac{\mathcal{B}(B_s \to \tau^+\tau^-)}{\mathcal{B}(B_s \to \mu^+\mu^-)} = \frac{m_\tau^2}{m_\mu^2}\sqrt{1-\frac{4m_\tau^2}{M_{B_s}^2}}\left(1-\frac{2m_\tau^2}{M_{B_s}^2}\right) = 166\,, \tag{4.14}$$

$$\frac{\mathcal{B}(B_d \to \tau^+\tau^-)}{\mathcal{B}(B_s \to \mu^+\mu^-)} = \frac{f_{B_d}^2 M_{B_d}^5 |V_{td}|^2 \tau_{B_d}}{f_{B_s}^2 M_{B_s}^5 |V_{ts}|^2 \tau_{B_s}}\frac{m_\tau^2}{m_\mu^2}\sqrt{1-\frac{4m_\tau^2}{M_{B_d}^2}}\left(1-\frac{2m_\tau^2}{M_{B_d}^2}\right) = 6.8 \pm 1.4\,. \tag{4.15}$$

By means of eq. (4.10), these allow us to set the following indirects limits:

$$\mathcal{B}(B_s \to \tau^+\tau^-) < 4.3 \times 10^{-4}\,, \qquad \mathcal{B}(B_d \to \tau^+\tau^-) \lesssim 2 \times 10^{-5}\,. \tag{4.16}$$

Unfortunately the present experimental information on the $\tau^+\tau^-$ modes is very poor: we are not aware of any progress with respect to the indirect bounds, at the per cent level, discussed in ref. [23]. The signature of these modes is certainly very difficult at hadron colliders, but significant improvements could be expected from B factories. Reaching a 10^{-5} sensitivity on the $B_d \to \tau^+\tau^-$ mode, B-factory experiments could compete with the present search for $B_s \to \mu^+\mu^-$ at Fermilab.

5. Conclusions

In this paper we have presented a general analysis of scalar FCNC amplitudes of down-type quarks in two classes of models with two Higgs doublets: the MSSM and the non-supersymmetric 2HDM of type II. We have shown that the $\tan\beta$ growth of these amplitudes is not simply determined by the number of Yukawa couplings appearing in the amplitudes. A crucial role is also played by the breaking of the $\mathrm{U}(1)_d$

symmetry that forbids the tree-level coupling of H_u to down-type quarks. Within the MSSM, the $\mathrm{U}(1)_d$ breaking induced by the $\widetilde{H}_u$–$\widetilde{H}_d$ mixing is not necessarily suppressed in the large-$\tan\beta$ limit, resulting in a potential $\tan^2\beta$ growth of the effective $\bar{d}_R^k d_L^j H_d$ vertex generated at one loop. This *anomalous* $\tan\beta$ behaviour is characteristic of the effective FCNC coupling of down-type quarks to neutral Higgses and it does not appear in irreducible $\Delta F = 2$ amplitudes at the one-loop level. On the other hand, because of the $1/\tan\beta$ suppression of the ordinary H_u–H_d mixing, this phenomenon does not arise at all in the non-supersymmetric model. As explicitly shown, in both cases leading $\tan\beta$ contributions to scalar amplitudes can efficiently be computed in the gaugeless limit of the models, or considering only the Higgs sector of the theory.

Despite the potential large enhancements (if $\tan\beta \gtrsim m_t/m_b$), non-standard scalar FCNC amplitudes are difficult to be identified experimentally. The rare dilepton decays $B_{s,d} \to \ell^+\ell^-$ offer an almost unique opportunity in this respect. As pointed out first in ref. [3], the $\tan^3\beta$ growth of the $b \to s\mu^+\mu^-$ scalar amplitude, within the MSSM, could lead to order-of-magnitude enhancements of the $B_s \to \mu^+\mu^-$ rate compared to SM expectations. We have explicitly checked that this statement remains true even when taking into account the existing constraints on $\Delta B = 2$ scalar operators. The latter includes two types of effects: i) the reducible two-loop contribution proportional to $y_b y_{s,d} \tan^4\beta$ discussed in ref. [6]; ii) an irreducible one-loop contribution proportional to $y_b^2 \tan^2\beta$. The two effects (comparable in size in the B_s–$\bar{B}_s$ case) are typically smaller than the SM amplitude and weakly constrained at present.

The order of magnitude enhancements of $B_{s,d} \to \ell^+\ell^-$ rates can occur only for $\tan\beta \gtrsim 30$, when higher-order $\tan\beta$-enhanced terms cannot be neglected. Following the approach of ref. [10], we have discussed how to control these terms beyond lowest order. Despite a drastic reduction of the one-loop result due to higher-order $\tan\beta$-enhanced terms (up to a factor 5 for $\tan\beta \sim 50$), the possibility that $\mathcal{B}(B_s \to \mu^+\mu^-)$ is just below its present experimental bound is still open. An evidence of $\mathcal{B}(B_s \to \mu^+\mu^-)$ at the 10^{-7} level (or above) would provide a strong support in favour of a supersymmetric explanation of the $(g-2)_\mu$ puzzle [22]; however, we have shown that the opposite is not true in the general MSSM framework: a sizeable supersymmetric contribution to $(g-2)_\mu$ does not necessarily imply observable consequences in $\mathcal{B}(B_s \to \mu^+\mu^-)$. Finally, we have emphasized the interest of $B_{s,d} \to \tau^+\tau^-$ modes in probing non-standard scalar FCNC amplitudes and, particularly, the importance to search for $B_d \to \tau^+\tau^-$ at B factories.

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A. Loop functions

$$
\begin{aligned}
S_0(x) &= \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3}{2}\frac{x^3 \log x}{(1-x)^3}\,, & S_0\left(\frac{m_t^2}{M_W^2}\right) &= 2.4 \pm 0.2\,,\\
f(x,y) &= \frac{1}{x-y}\left[\frac{x\log x}{1-x} - \frac{y \log y}{1-y}\right]\,, & f(1,1) &= \frac{1}{2}\,,\\
F(x,y,z) &= \frac{x \log x}{(x-1)(x-y)(x-z)}\,, & &\\
L_1(x,y,z) &= xy[F(x,y,z) + F(y,z,x) + F(z,x,y)]\,, & L_1(1,1,1) &= -\frac{1}{6}\,,\\
L_2(x,y,z) &= xy[xF(x,y,z) + yF(y,z,x) + zF(z,x,y)]\,, & L_2(1,1,1) &= \frac{1}{3}\,,\\
L_3(x,y,z) &= \frac{1}{xy}L_2(x,y,z)\,, & &\\
L_4(x,y,z) &= \frac{1}{xy}L_1(x,y,z)\,, & &\\
G_2(y) &= \frac{1 - 9y - 9y^2 + 17y^3 - 6y^2(3+y)\log y}{6(y-1)^5}\,, & G_2(1) &= -\frac{1}{20}\,,
\end{aligned}
$$

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